Dialing Habits of Telephone Customers

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This paper considers the behavior of customers waiting to dial calls, when dial tone is delayed. Tests were made in a panel dial central office, from which were determined: relationship between load carried by a group of line finders and the resultant dial tone delay; measures, by classes of service, of the magnitude of the generalized trunking formula's "j" factor describing the degree to which customers wait when dial tone is delayed; comparisons of observed and theoretical distributions of the number of simultaneous calls on line finder groups; and statistical accounts of the actions of customers when dial tone is delayed.

Following World War II the conversion of great quantities of manual telephone equipment to dial, and the addition of large numbers of new telephones, mostly dial, in the Bell System has directed increasing attention to those service problems peculiar to automatic operation. These problems concern chiefly the provision of adequate amounts of equipment to give satisfactory service at all times. One of the important factors affecting the amount of equipment needed is the action of the customers themselves when their calls are momentarily blocked due to these equipment shortages. The actions of subscribers whose calls are blocked due to a shortage of trunk equipment have been reported previously.[1] This paper considers the behavior of subscribers, waiting to dial calls, when dial tone is delayed.

During 1949, Bell Telephone Laboratories conducted a series of tests at the New York Telephone Company's Sterling-3 panel dial central office in Brooklyn, N. Y., with the object of increasing the knowledge available regarding subscribers' actions and their effects when dial tone is delayed.

The following principal results were obtained from the Sterling-3 tests:

1. The relationship between the load carried by a group of line finders and the resultant dial tone delay.

2. Measures, by classes of service, of the magnitude of the general-

[1] Charles Clos, "An Aspect of the Dialing Behavior of Subscribers and Its Effect on the Trunk Plant," *Bell System Tech. J.*, **27**, July 1948.

ized trunking formula's "j" factor describing the degree to which customers wait when dial tone is not immediate.[2]

3. Comparisons of observed and theoretical distributions of the numbers of simultaneous calls on the line finder groups.

4. Statistical accounts of the actions of subscribers when dial tone is delayed.

GENERAL PLAN OF THE TESTS

The general plan of the tests was developed around two specially constructed devices:

1. A 100-pen recorder capable of recording observations continuously for two hours and sensitive enough to record individual dial pulses.

2. A speed of dial tone measuring set comprising a means for manually originating test calls, and an electric timer which automatically stopped when dial tone was received.

Tests were conducted on a weekly schedule, one or two line finder groups being studied each week. Two 100-pen recorder tapes were run daily. The first was run from 10 A.M. to noon for message rate individual, flat rate individual and message rate two-party classes of service to include the morning busy periods for these customers. For coin customers the first tape was run for two hours during the noon coin busy period. The second tape was run in the afternoon from 2 to 4 P.M. for all classes of service except coin. For the coin class the second tape was run for two hours during the early evening coin busy period. Three message rate two-party tapes were run in the early evening busy period, and two coin tapes were run in the afternoons when World Series baseball games were being played in Brooklyn. A summary of the number of line finder groups observed, the number of tapes taken, the number of line finders made available and the maximum per cent dial tone delays over three seconds are given in Table I.

Except for the morning runs on the message rate individual line groups where an effort was made to maintain a fixed number of twenty line finders throughout, the number of line finders made available was selected by close observation of the flow of traffic. All studies were by half hours during which the number of line finders was held constant as far as possible. At the end of a half-hour period the number of line

[2] This formula for both finite and infinite sources was developed by R. I. Wilkinson in 1930, and appeared in the 1936 Bell Telephone Laboratories Out-of-Hour Course "The Theory of Probability as Applied to Telephone Trunking Problems." This formula for infinite sources was also developed by Conny Palm and appeared in "Etude des delais d'attente" in Erickson Technics—No. 2—1937.

finders left in service was adjusted in order to obtain a reasonable number of dial tone delays in the next half hour without producing a severe reaction from the subscribers served by the line groups under study.

The data recorded on the tapes showed continuously the busy or idle conditions of certain circuits associated with the line groups under study. In some cases the receipt of dial pulses and the operation of registers were also recorded. These circuits included line finders, a few subscriber lines, trip circuit sub-groups, the all trunks busy register, the peg count register and the speed of dial tone measuring device. In addition the busy and idle conditions of a sample of senders was observed in order to note the general load level on tbe senders.

TABLE I

	Number of Line Finder Groups Observed	Number of Tapes	Number of Line Finders Made Available	Maximum Percent Dial Tone Delays Over 3 Seconds
Message rate individual	11			
Morning tapes		25	19 to 20	53.3
Afternoon tapes		29	10 to 15	55.6
Message rate two-party	3			
Morning tapes		8	18 to 19	40.0
Afternoon tapes		12	10 to 12	88.6
Evening tapes		3	19 to 21	71.2
Flat rate individual	2			
Morning tapes		9	14 to 33	48.9
Afternoon tapes		10	10 to 17	35.6
Coin	3			
Morning tapes		18	30 to 39	71.1
Afternoon tapes		2	30 to 39	35.6
Evening tapes		10	25 to 39	68.9

RELATIONSHIP BETWEEN LOAD CARRIED AND PER CENT DIAL TONE DELAY

One of the principal objectives of these tests was to establisb as far as possible the relationship between the average load carried by a line-finder group and the corresponding dial tone service when tbere is a shortage of line finders but not of senders.[3] The average load carried was obtained by making a switch count every thirty seconds of the number of line finders busy as indicated by the 100-pen recorder tape. The dial tone tests were made with the speed of dial tone measuring device. Forty-five dial tone tests were made each half hour for each

[3] The restriction of avoiding dial-tone delays due to a sender shortage was to eliminate a factor external to the line-finder group and to make the results of the tests applicable to both common-control and non-common-control type dial systems.

line group studied. Additional dial tone tests were made on all other line groups in the office as a check that the delays experienced on the line groups under study were not due to a sender shortage. The sender data on the tapes were also used for this purpose. Figs. 1(a) to 1(d), and 2(a) to 2(d), inclusive, show for various amounts of load carried, the per cent of dial tone tests encountering delays greater than three and greater than ten seconds for half-hour study periods for the most frequent number of line finders in the tests for each class of service.

Plotted on each of these figures is a theoretical fitting dial tone tester delay curve computed for the indicated dial tone delays and for the following j factor values in the generalized trunking formula, determined in a manner to be explained later:

Class of Service	j factor
MRI	6.6
MR 2-party	5.8
FRI	6.5
Coin	2.1

To indicate the effect of varying j, several curves have been added to Fig. 1(a). Selections of curves for $j = 0$, 1 and ∞ (which correspond to the three commonly used infinite source congestion formulae, Erlang C, Poisson and Erlang B when adapted to the tester's delay problem) are shown. It is clear that with the wide differences in delays which they give for specified loads carried, it is highly desirable to select that j formula for engineering use which most nearly describes the customer actions in any situation being dealt with. In the field of curves shown on Fig. 1(a), the one labelled $j = 6.6$ was derived in a logical manner from the data, and shows an agreeably satisfactory fit. For example, during a heavy load period when, say, 20 per cent of a dial tone tester's calls are meeting delays greater than 3 seconds, an actual average load of 16.6 erlangs (as shown by the $j = 6.6$ curve) would likely be carried. (Load in erlangs equals average number of simultaneous calls.) The Erlang C ($j = 0$) and Poisson ($j = 1$) theories would indicate the presence of loads of 15.6 and 16.0 erlangs, respectively, figures clearly too small for the circumstances shown by the data of Fig. 1(a). On the other hand, use of the Erlang B ($j = \infty$) theory would predict a considerably larger load carried, about 17.9 erlangs, than one would probably be justified in assuming here for engineering purposes.

By grouping the dial tone delay data by bands of load carried, relationships of per cent of test calls encountering varying dial tone delays

up to twelve seconds were obtained. These data are shown on Figs. 3 to 6. Fig. 3 is for the message rate individual class of service with twenty line finders available. The data on this figure correspond to those on Figs. 1(a) and 1(b). Fig. 4 is for the message rate two-party class of service with ten line finders and corresponds to Figs. 1(c) and 1(d). Fig. 5 is for the flat rate class of service with ten line finders and cor-

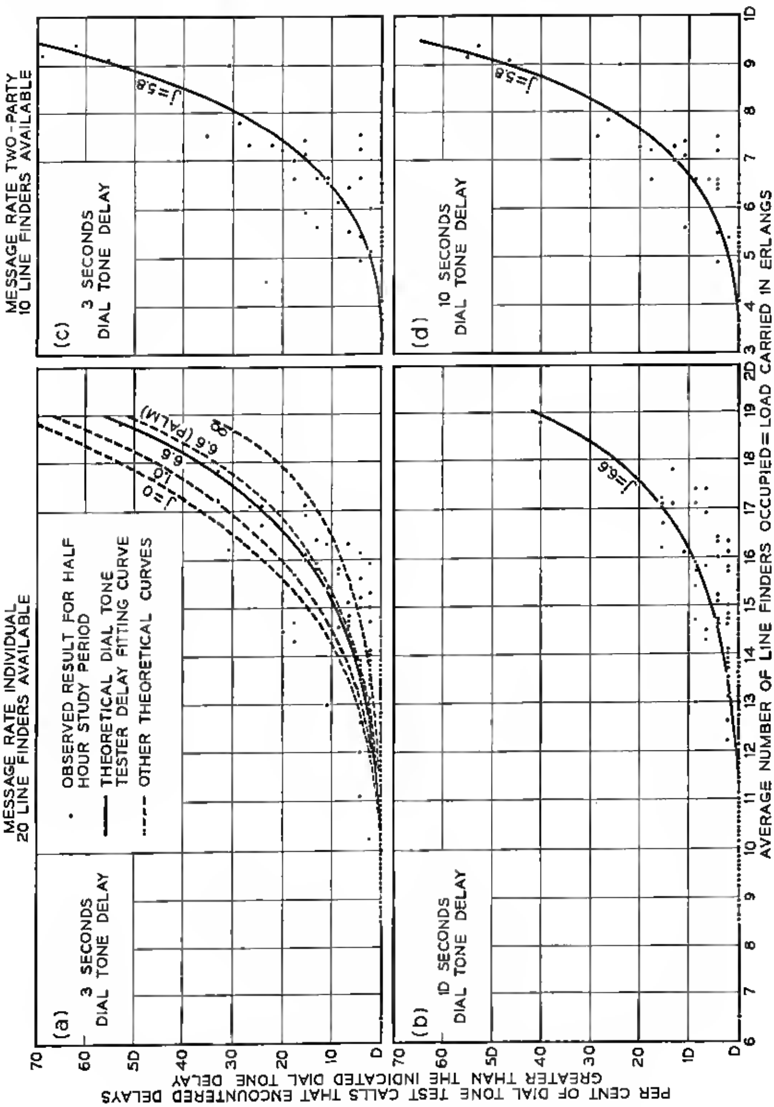

Fig. 1.—Results of dial tone tests.

responds to Figs. 2(a) and 2(b). Fig. 6 is for the coin class of service with 34 line finders and corresponds to Figs. 2(c) and 2(d).

Plotted on Figs. 3 to 6 are theoretical fitting dial tone tester delay curves, curves A, determined by means of the following formulae:

1. The generalized trunking formula for determining the proportion of calls that encounter congestion, i.e., find all line finders busy.

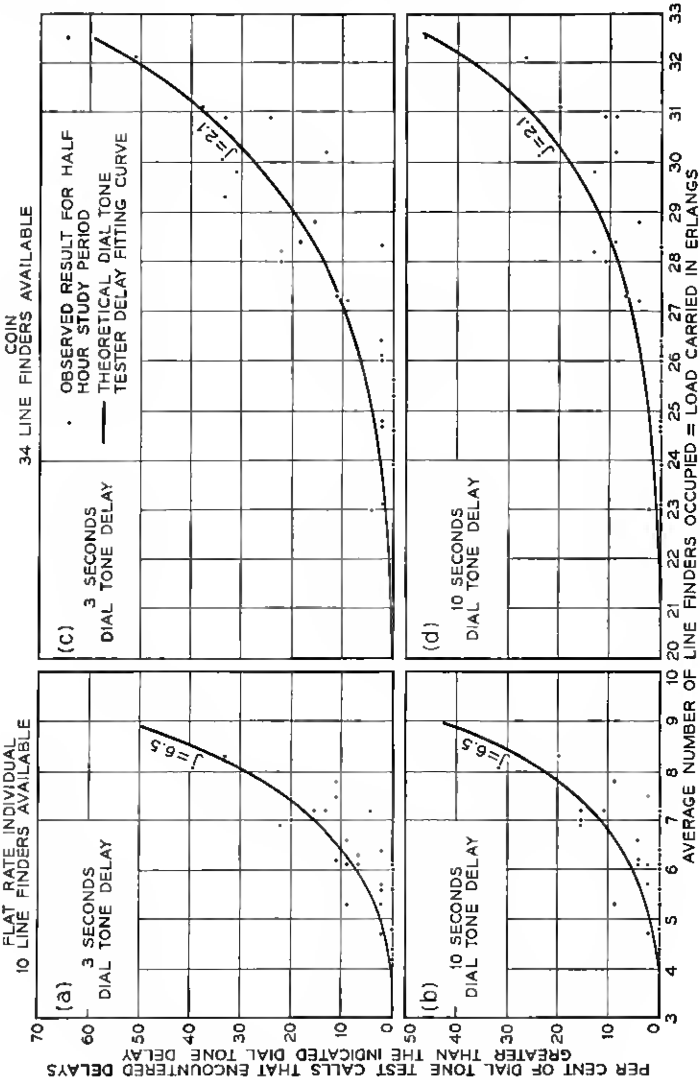

Fig. 2—Results of dial tone tests.

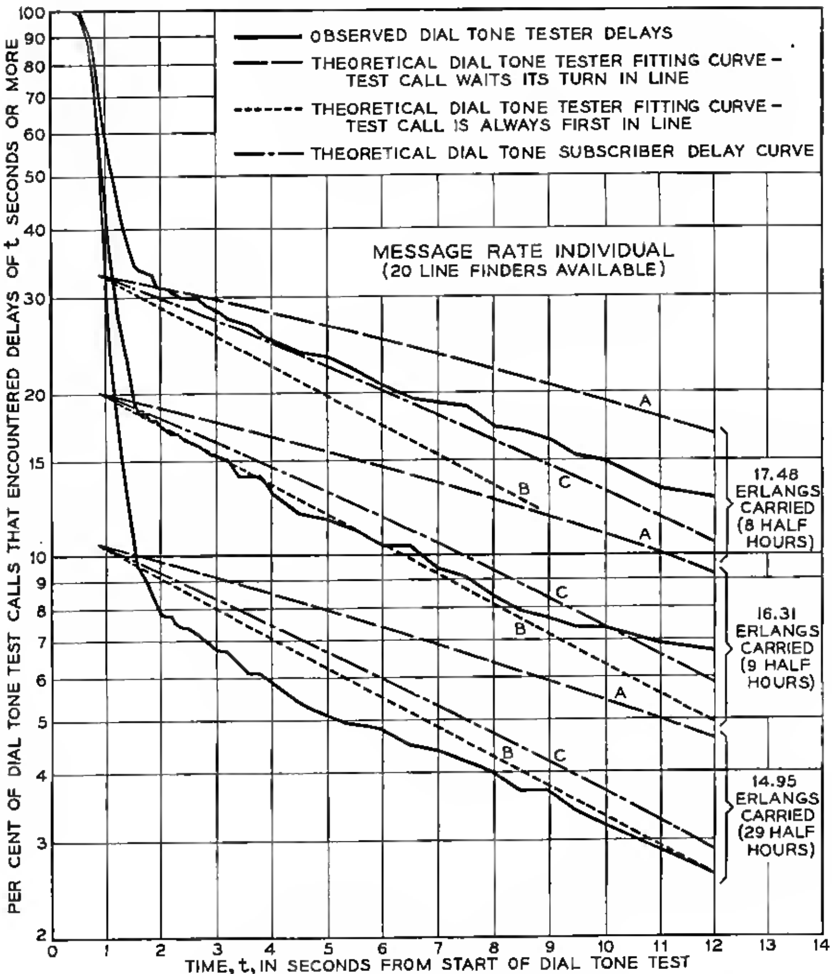

Fig. 3—Results of dial tone tests.

2. A delay formula[4] for determining the proportion of those test calls encountering congestion which will have a delay in obtaining dial tone of at least time t.

Some of the theoretical aspects of these formulae are considered in the two sections that follow.

GENERALIZED TRUNKING FORMULA

The generalized trunking formula combines in one expression the various assumptions underlying the Erlang B, Poisson, and Erlang C

[4] A development by John Riordan paralleling that of Conny Palm's.

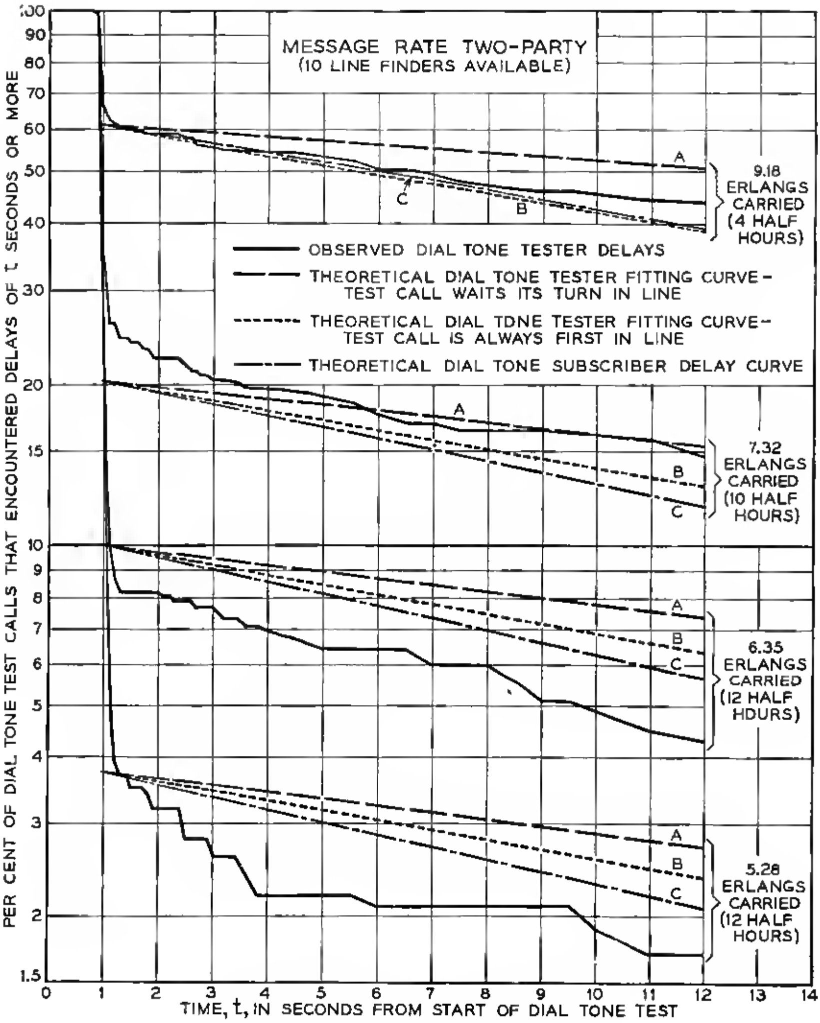

Fig. 4—Results of dial tone tests.

trunking formulae and a large field of intermediate assumptions regarding the disposition of calls which fail to obtain immediate service. These assumptions are given in Table II on page 42.

One method for developing the generalized trunking formula[5] is to consider the probabilities of the existence of certain states and to determine the number of transitions during some convenient interval of

[5] In this article only the unlimited sources trunking formula is considered.

time from one state to another. By equating certain of these transitions, a series of simultaneous equations evolves, which when solved yields one overall expression. Of interest is the development of the transition equations. Thus for a case of c line finders arranged in a simple group, let $f(x)$ represent the probability that x (where $x < c$) line finders are occupied and $f(x + 1)$ represent the probability that $x + 1$ line finders are occupied. Let n be the average number of calls offered to the line finders during a long interval of time, T. Let T be the unit of time and h be the average holding time per call measured in terms of T. Over a

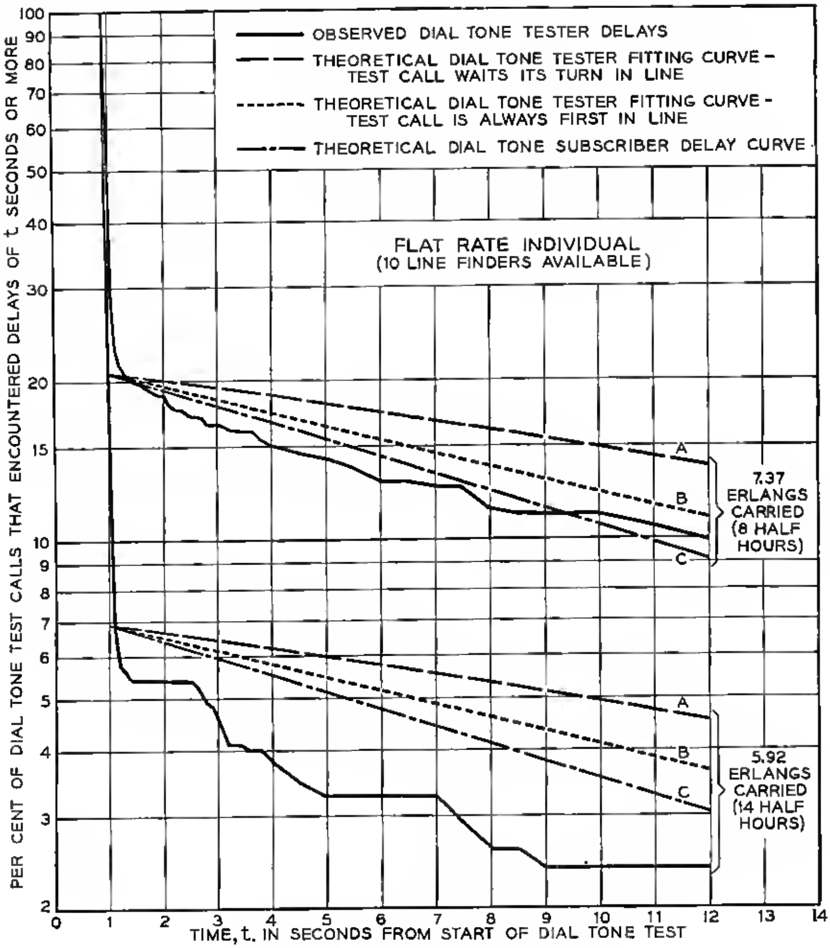

Fig. 5—Results of dial tone tests.

period T the number of transits from state x to state $x + 1$ must equal (or differ by no more than one) the number of transits in the reverse direction. That is:

$$nf(x) = \frac{x + 1}{h} f(x + 1) \tag{1}$$

It may be noted that $nh = a$, where a is the average offered traffic load in erlangs.

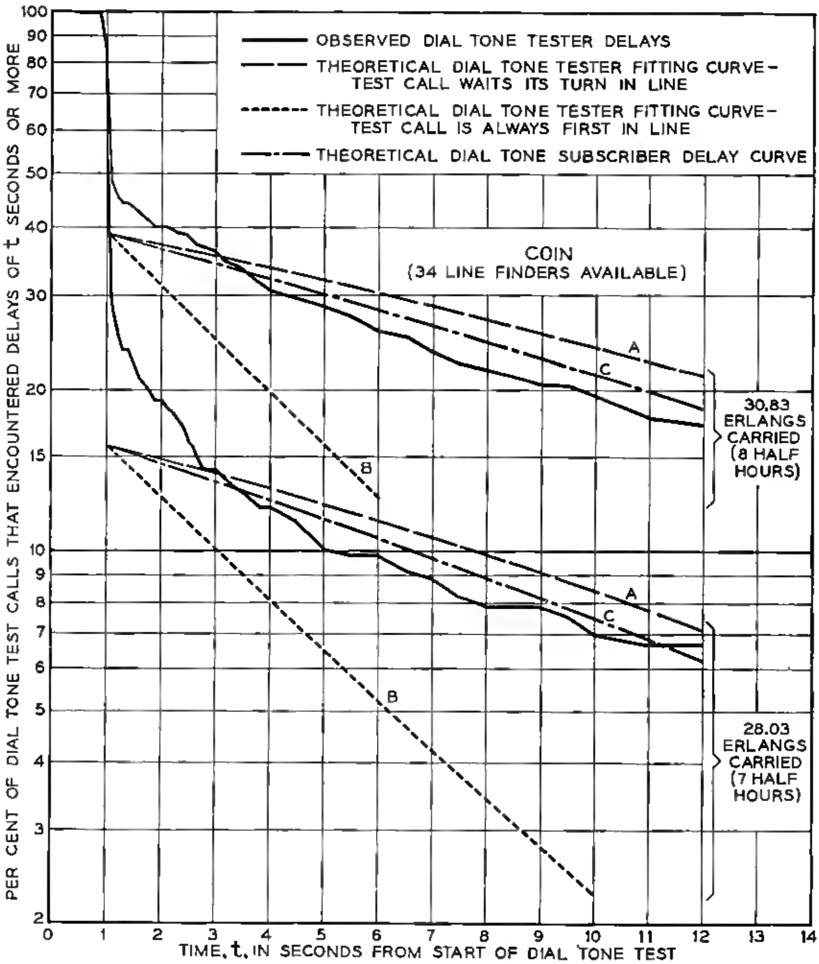

Fig. 6—Results of dial tone tests.

TABLE II

Formula	Assumption Concerning the Disposal of Calls that do not Obtain a Line Finder Immediately
Generalized	Waiting calls are cleared out at a rate j times the rate at which calls are terminated when served by the line finders.
Erlang B	Calls are cleared out of the system immediately, that is no calls wait ($j = \infty$).
Poisson	Waiting calls are cleared out at a rate equal to that with which calls are terminated when served by the line finders ($j = 1$).
Erlang C	Calls wait until served ($j = 0$).

When $x \geqq c$ a new situation is encountered. "c" calls are engaged in conversation and $x - c$ calls are waiting for service. If the waiting calls are forced to wait for an unduly long period of time so that in effect they are being denied service, it can be expected that they will wait for some average period, say H, and then abandon their attempts. On this basis the corresponding equation is:

$$nf(x) = \frac{c}{h} f(x + 1) + \frac{(x + 1 - c)}{H} f(x + 1) \tag{2}$$

It has been assumed in the above equations that the distribution of the holding times is exponential, an assumption which is found in most local systems to be reasonably justified. The distribution of the waiting times is also taken to be exponential. By introducing a factor j, where $j = h/H$, equation (2) can be written in the simpler form:

$$af(x) = [c + j(x + 1 - c)]f(x + 1) \tag{3}$$

Solving this system of simultaneous equations, we obtain:
when $x < c$,

$$f(x) = \frac{a^x}{x!} f(0) \tag{4}$$

when $x \geqq c$,

$$f(x) = \frac{a^x}{c!(c + j)(c + 2j) \cdots\cdots\cdots [c + (x - c)j]} f(0) \tag{5}$$

where

$$f(0) = \left(\sum_{x=0}^{x=c} \frac{a^x}{x!} + \sum_{x=c+1}^{x=\infty} \frac{a^x}{c!(c + j)(c + 2j) \cdots [c + (x - c)j]} \right)^{-1} \tag{6}$$

The probability of a call encountering congestion, which is equivalent to the probability of a call having a delay greater than zero units of time is:

$$P(>0) = \sum_{x=c}^{x=\infty} f(x) \tag{7}$$

DELAY FORMULA FOR THE DIAL TONE TESTER

The probability that a dial tone test call encounters congestion is given by expression (7). Once a test call has encountered congestion it will experience a delay depending upon a number of variables. The assumptions underlying the dial tone tester formula are:

1. A dial tone test call when encountering a delay waits until served.
2. A dial tone test call does not add to the load offered and carried by the line finders.
3. Upon encountering a delay, a dial tone test call is served in the order of its arrival with respect to all other waiting calls. For example, if the test call finds three other calls waiting, it waits fourth in line.

Under the third assumption as calls drop out, due to conversations terminating on the occupied line finders or due to waiting calls abandoning their requests for service, the test call advances from an initial position of say fourth in line to third in line, then to second, then to first in line, and finally is served. The overall delay distribution of the test calls depends therefore upon the number of calls they find waiting ahead of them. The delay distribution for each such number must be weighted by the probability of its occurrence in order to obtain the overall distribution. The delay distribution for a test call which finds zero calls waiting is:

$$p_0(>t) = \exp(-ct/jH) \tag{8}$$

The probability is $f(c)$ that a call made at random will find all line finders busy with no calls waiting. Hence the weighted delay distribution $P_0(>t)$, is:

$$P_0(>t) = f(c)p_0(>t) = f(c)\exp(-ct/jH) \tag{9}$$

The delay distribution for a test call which finds one call waiting ahead of it is:

$$p_1(>t) = [1 + c/j - (c/j)\exp(-t/H)]\exp(-ct/jH) \tag{10}$$

The probability is $f(c + 1)$ that a call made at random will find all

line finders busy and one call waiting. Hence the weighted delay distribution, $P_1(>t)$, is:

$$P_1(>t) = f(c+1)[1 + c/j - (c/j) \exp(-t/H)] \exp(-ct/jH) \quad (11)$$

In the general case $p_n(>t)$ is given by the following formula:

$$p_n(>t) = F_{n+1}(t) \exp(t/H) \quad (12)$$

where $F_{n+1}(t)$ is given by Conny Palm.[6] The over-all delay distribution is then:

$$P(>t) = P_0(>t) + P_1(>t) + P_2(>t) + \cdots \quad (13)$$

By making appropriate substitutions and summing the result, expression (13) becomes:

$$P(>t) = \frac{a^c}{c!} f(0) \left[1 + \frac{a \exp(-t/H)}{c+j} + \frac{a^2 \exp(-2t/H)}{(c+j)(c+2j)} + \cdots \right] \exp\left[-ct/jH + (a/j)[1 - \exp(-t/H)] \right] \quad (14)$$

Expression (14) is equivalent to that of Riordan involving two incomplete gamma functions as follows:

$$P(>t) = P(>0) \frac{\gamma[c/j, (a/j) \exp(-t/H)]}{\gamma(c/j, a/j)} \quad (15)$$

where the incomplete gamma function,

$$\gamma(N, x) = \int_0^x x^{N-1} e^{-x} \, dx \quad (16)$$

The theoretical dial tone tester delay curves shown on Figs. 1(a) to 1(d), 2(a) to 2(d), and 3 to 6 were computed from expression (14), using the following values of j and H for the classes of service studied, these values being determined in a manner explained later:

Class of Service	j factor	H
MRI	6.6	24 seconds
MR 2-party	5.8	42 seconds
FRI	6.5	27 seconds
Coin	2.1	74 seconds

On Figs. 1(a), 1(c), 2(a), and 2(c), which show the per cent of dial tone tests encountering delays greater than three seconds for various amounts

[6] Equation 53, loc. cit.

of load carried, it may be noted that most of the theoretical dial tone tester delay curves are in close agreement with the observed data, with a tendency perhaps to be slightly high. On Figs. 1(b), 1(d), 2(b), and 2(d), which are for dial tone delays greater than ten seconds, it may be noted that the theoretical curves have a slightly stronger tendency to lie on the high side of the observed data. On Figs. 3 to 6 the theoretical dial tone tester delay, curves A, again lie in the proximity of the curves of the observed data, with a tendency to lie higher than these latter curves, especially at the ends where the dial tone delays are greatest. Among the factors which account for this discrepancy are:

1. A feature is present in panel line finder circuits for momentarily releasing trip circuits with waiting calls to prevent the orphaning of calls under certain trouble conditions. The release occurs after a call has been waiting from 5 to 12 seconds and reoccurs every 7 seconds thereafter. When such a release occurs the call yields whatever waiting preference it may have had to a subsequently placed call which is not yet affected by such a release. The dial tone test calls did not wait beyond 12 seconds. Hence for these test calls there was only one possibility of such a release and for many of them the release occurred near the end of their waiting period. Hence they were more likely to gain preference over ether calls than to lose their preference.

2. Subscribers while waiting for dial tone frequently become impatient and proceed to flash (move their switchhook up and down). While flashing, a subscriber may lose preference to a subsequently placed test call (the latter of course does not flash).

3. Many subscribers fail to observe dial tone and proceed to dial. During such dialing, a subscriber may lose preference to a subsequently placed test call.

4. Line finders serve a large proportion of call attempts of short holding time whose presence may militate against the occurrence of the longer delays. In connection with the measurement of the j factor, the following proportions of call attempts and average holding times were noted on which no dialing occurred or where no more than two digits were dialed.

Class of Service	Proportion of Attempts with No Peg Counts	Average Holding Time
MRI	35.2%*	4.4 seconds*
MR 2-Pty.	33.3%*	5.8 seconds*
FRI	25.1%	5.8 seconds
Coin	9.8%	8.3 seconds

* Partly estimated

The individual contributions of these four factors to discrepancies between theory and observation are not easy to assess. The first three explain a tendency for test calls to get ahead of calls already waiting for dial tone. On Figs. 3 to 6, inclusive, additional theoretical dial tone delay curves, curves B, for the case where a dial tone tester always gets first in line are shown. Even these curves tend to lie above the curves of the observed data on Figs. 4 and 5 where ten line finders were available; they more nearly agree with the observed data on Fig. 3 where twenty line finders were available, and they lie below the observed data on Fig. 6 where 34 line finders were available. This is an indication of the fact that with higher traffic loads (which occurred on the larger line finder groups) a test call will encounter more competition from other calls and therefore will have a lesser chance of gaining precedence over all of the other calls. The fourth factor indicates that the call attempts served on line finders consist of two distinct holding time universes and not just one, as was assumed in the development of the dial tone tester formula. The effect of the presence of both a short and long holding time universe of calls would be to introduce a change of slope in the delay curves which may be seen in Figs. 3 to 6 to be at about $t = 4$ seconds. There is reason to believe that the same cause may have been responsible for the tendency of the observed delay curves to fall away from the theoretical at the lower levels of load carried.

Due to the reasons given above and to the fact that the dial tone delay observations were made by the test call method, the above results may not directly describe service from the customer's point of view. Conny Palm has developed the following formula which gives a slightly different measure of customers' dial tone service. It indicates the proportion of calls which have neither received dial tone nor have dropped out at time t.

$$P(>t) = P(>0) \frac{\gamma[c/j, (a/j) \exp(-t/H)]}{\gamma(c/j, a/j)} \exp(-t/H) \qquad (17)$$

Curves for this formula are shown plotted on Fig. 1(a) and at C on Figs. 3 to 6. They are quite close in many cases to the observed dial tone tester results. It would appear that a sufficiently good estimate of the customer's dial tone service, whatever its precise definition, can be obtained by the dial tone tester method.

Recently revised tables for the capacity of step-by-step line finders have been published for Bell System use based on Palm's formula using a factor of $j = 5$. This was selected as being slightly conservative for

most applications after reviewing the above Sterling-3 results and other line finder data collected in step-by-step offices.

MEASUREMENT OF THE j FACTOR BY CLASSES OF SERVICE

As indicated previously, the data recorded on the tapes showed the states of being busy or idle and of changes in these states for line finders and the associated trip circuits. A fully equipped line finder group of 400 lines has ten trip circuits each of which serves two sub-groups of twenty subscriber lines in the following manner. When a line originates a call its line relay is operated. This causes a ground to appear on a lead which is common to all twenty line relays in the sub-group and starts a line finder hunting for the calling subscriber's line. As soon as this hunt is completed the cutoff relay associated with the calling line operates and disconnects the line relay, removing the ground (unless, of course, another line in the sub-group has originated a call in the meantime). During periods of overload when line finders are not immediately available, the ground due to a single subscriber will persist until:

1. A line finder is obtained, or
2. The subscriber abandons the attempt, or
3. The subscriber receives an incoming call which operates the cutoff relay.

The twenty leads from the trip circuit sub-groups were brought out to the pen recorder and a record taken of the grounds that occurred on each lead. Except for the possibility that more than one subscriber is waiting for service at the same time on a given trip circuit sub-group, the record of the occurrences of the grounds gives a substantially accurate[7] record of the demands for service and of the number of calls waiting for service. Hence an analysis of the events occurring on the trip circuit sub-groups and on the line finders as recorded on the tapes gives a means for determining H. The quantity H was introduced in equation (2) in the term

$$\frac{(x + 1 - c)}{H} f(x + 1) \tag{18}$$

For convenience in the ensuing discussion this term will be replaced by

[7] To obtain absolute accuracy would require the use of a pen recorder with one pen for each of the 400 subscribers served on a line finder group plus one for each line finder.

the equivalent expression:

$$N_y = \frac{y}{H} f(z + y) \tag{19}$$

where N_y = The average number of waiting calls that drop out per unit of time during the state $(z + y)$.

y = The number of waiting calls.

z = The number of line finders occupied with calls.

$1/H$ = A measure of the rate at which calls tend to abandon waiting.

$f(z + y)$ = The proportion of time that the state $(z + y)$ exists.

On the tapes we can measure $f(z + y)$ and count N_y. Hence H can be determined. The result is a statistical quantity subject to many chance factors. In the actual analysis of a tape, the composite average value of H was determined for all possible observed states where calls were waiting. By an analogous process, the composite average value of h for all possible observed states where calls were being served by line finders was determined. Also as a side computation, a composite average value of h' for calls that were served by line finders but for which no peg counts were scored was determined. This value of h' is included in h on the basis that data for engineering line finders consist of estimated calls based on peg counts and of holding times which include an allowance for these short holding time calls. The average values of H, h and of the j factor for the four classes of service studied are given in Table III.

The results for H by individual half hours and by various percentages of dial tone delays greater than three seconds are shown on Figs. 7(a) to 7(d) respectively for the four classes of service. On some of these figures an upward bulge may be noted in the center. This is not considered to be characteristic of the habits of the subscribers but is the overall effect resulting from a number of arbitrary rules followed in making the analysis in order to simplify the work and to offset par-

TABLE III

	Average Values in Seconds		$j = h/H$
	H	h	
Message rate individual	24	159	6.6
Message rate two-party	42	243	5.8
Flat rate individual	27	176	6.5
Coin	74	153	2.1

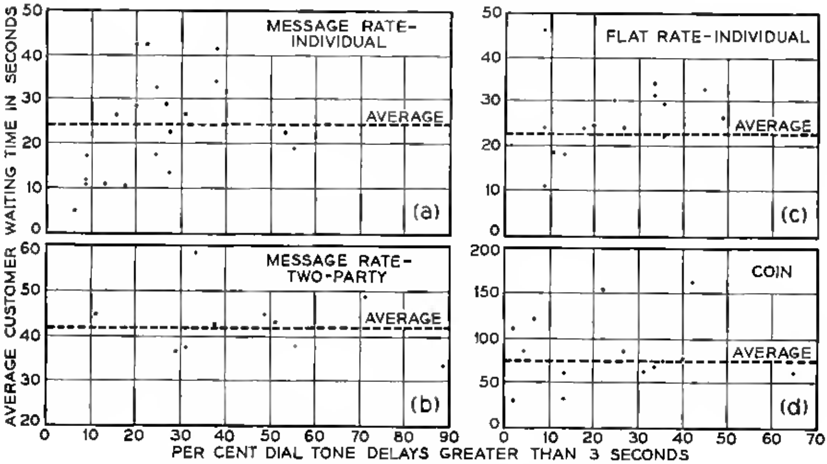

Fig. 7—Average customer waiting time (H).

tially the effect of occasionally having two or more calls waiting on one trip circuit sub-group. The rules and the reasons for them will be described with the aid of Fig. 8, which shows a hypothetical section of one of the tapes. The rules were as follows:

1. Initial Overlap

Referring to Fig. 8, at t_1 a subscriber has initiated a request for service. At t_2 a line finder rises to serve the subscriber. At t_3 the subscriber receives service. This case is typical of a subscriber receiving prompt dial tone service.

The span from t_1 to t_2 was difficult to measure accurately because, for the usual case, it was about the same as the maximum error due to misalignment of the recorder pens. It was not measured unless the combined span from t_1 to t_3 exceeded one second.

The span from t_2 to t_3 involves an overlap, it represents a period when a line finder is busy hunting for the terminal of the subscriber who originated the request for service. It also represents a period when a subscriber is waiting for service. In the analysis this span was treated as a case where a line finder was busy with a call and not as a call waiting for service.

If the span from t_2 to t_3 and all similar cases had been treated as calls waiting for service and if in addition all spans from t_1 to t_2 which were not measured had also been treated as calls waiting for service, the average values for H would have increased slightly for each class of service.

2. Three Second Rule for the Bridging of Calls

Referring to Fig. 8, again, at t_5 a request for service is originated on trip circuit 5 and at t_7 this request is withdrawn. At t_{10} apparently a new request for service is initiated which is then withdrawn at t_{11}. From manual service observations it is found that suhscribers often flash when dial tone is slow. A few pens were used to observe individual suhscrihers, and Fig. 9 shows a case where a subscriher made several flashes when his tone was slow. When a subscriber flashes it appears as though he

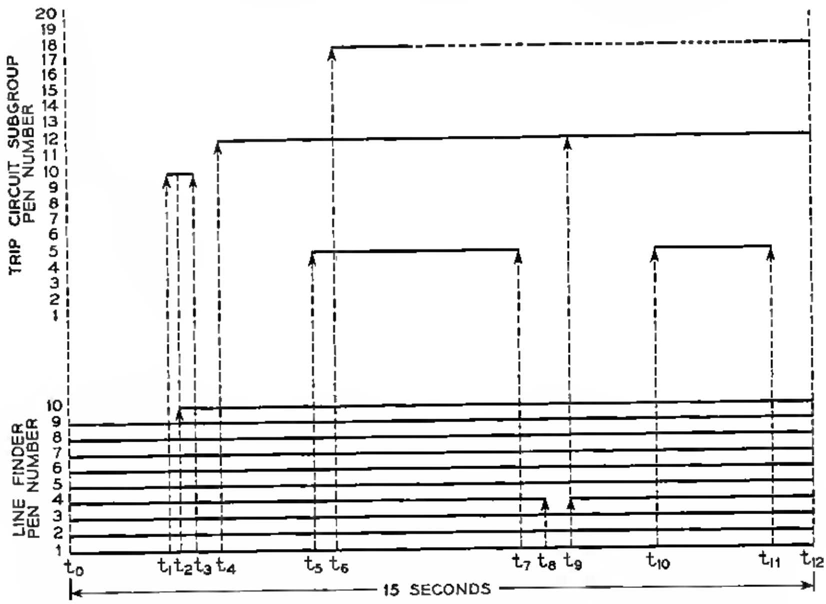

Fig. 8—Section of a hypothetical tape showing activities on trip circuit subgroups and on line finders.

were making several hids for service. Actually he is making only one real hid. On the trip circuit suh-group pens it was generally impossible to distinguish between flashes and requests for service hy two or more subscribers. To resolve this problem many observations of suhscriber lines recorded on the tapes were examined from which it was concluded that no great error would result if a break in the demand for service on a trip circuit suh-group of less than three seconds were considered as a flash aud was to he hridged, and a hreak greater than three seconds was to be considered as the termination of one call attempt and the start of another

3. Treatment of Cases Where Two or More Calls Were Found to be Waiting on One Trip Circuit Sub-Group

The occurrence of several calls waiting on one trip circuit was occasionally noted in the analysis. Referring to Fig. 8, a case is shown on trip circuit sub-group 12. At t_9 a line finder is seized. Trip circuit sub-group 18 shows that a subscriber is dialing before tone. The appearance of dial pulses on this trip circuit indicates that only one subscriber is demanding service otherwise the dialing would not show. Trip circuit sub-group 12 however appears to have two or more requests for service. One of these requests for service began at t_4. The start of the second request occurred somewhere between t_4 and t_9, perhaps half-way between. At t_9, one of the requests was served by a line finder. To simplify the handling of such cases, the assumption was made that the first attempt started at t_4 and ended at t_9 and the second request started

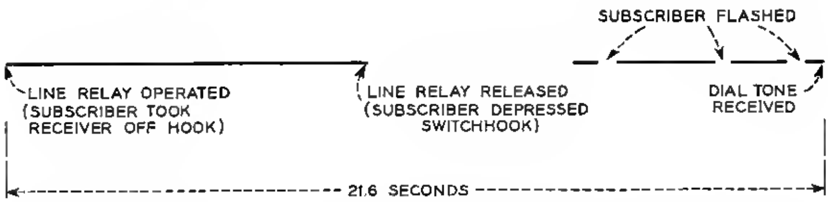

Fig. 9—Example of a customer flashing for dial tone (Tape made October 18).

at t_9. The effect of this is to understate by an indeterminate amount the average value of H for each class of service. This understatement should be noticeable for the higher degrees of overload because the occurrence of several calls on one trip circuit sub-group is then most likely to occur.

The effect of several calls simultaneously waiting in a trip circuit sub-group and of one or more calls dropping out is to overstate the magnitude of H. For instance if two simultaneous call attempts of five seconds each overlap for one second and both attempts are abandoned, the apparent average waiting period is nine seconds, whereas it should be five seconds. It is believed that the above three rules tend to create understatements which roughly balance this type of overstatement.

DISTRIBUTION CURVES OF SIMULTANEOUS CALLS

The detailed analysis of the tapes provided distributions of simultaneous calls. For each class of service studied these distributions can be compared with theoretical distributions derived from the generalized trunking formula using the j factors developed in the analysis. Several

such comparisons are shown on Figs. 10 and 11. Tbe agreement is quite good in most cases.

SUBSCRIBER DIALING HABITS AS OBSERVED WITH A MONITORING CIRCUIT ON A SENDER WITH INDUCED DIAL TONE DELAYS[8]

As a separate study a series of tests was made by means of a monitoring circuit on one of the senders serving in common the subscribers in the Sterling-3 and Main 2 central offices, for the purpose of obtaining further information on subscriber dialing characteristics under overload conditions. A large amount of data was collected on the time intervals from the seizure of the sender to the first action taken by subscribers when encountering dial tone delays, the latter being introduced under the control of tbe observer.

The monitoring circuit was wired to a particular sender in a group of 100 serving all classes of subscribers. When the circuit was in use, the only irregularity introduced was that the dial tone could be delayed even though tbe sender was actually available to the subscriber. The delay did not affect the sender in its functions if the subscriber elected to dial before tone.

The sender monitoring circuit provided the following four features:

1. A receiver was bridged across the tip and ring leads in the sender so that an observer could hear certain actions taken by a subscriber connected to the sender. The sender was of course disconnected before conversation.

2. The observer was able to preselect one of several intervals by which dial tone was delayed on successive calls served by the sender. This was accomplished with a capacitance-resistance-vacuum tube circuit.

3. By means of a timer which started when the sender was seized, the observer was enabled to note elapsed time intervals to the occurrence of the various actions of the subscribers. Tbe reading of the time of the first action of a subscriber had to be made when the second band was in motion, which introduced certain errors later to be discussed.

4. By means of colored lamps the observer was able to classify all calls observed as being message rate, flat rate or coin.

During the sender dial tone delay tests, observations were made only during the afternoons when the flow of traffic was light and tbe probability of a subscriber obtaining a delay before reaching the sender was a minimum.

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[8] Based on an unpublished report by W. A. Reenstra.

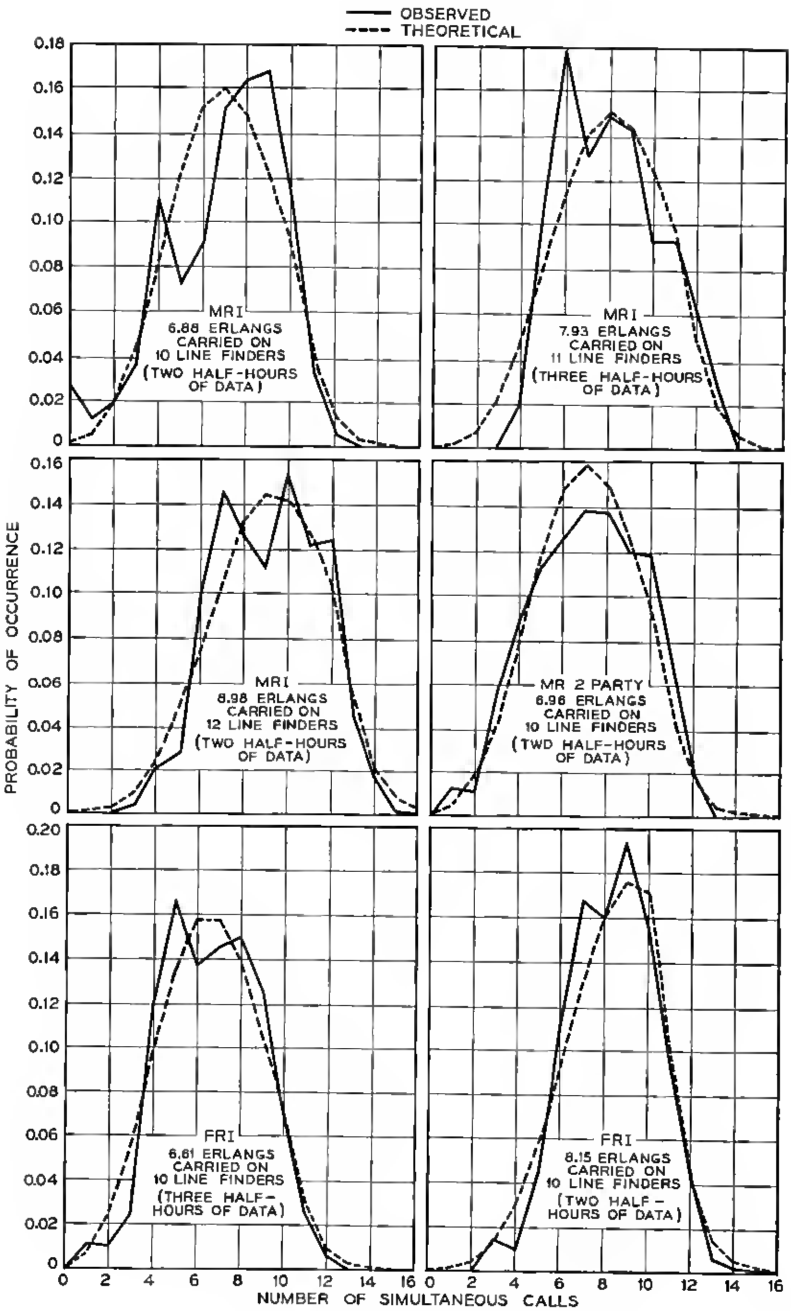

Fig. 10—Distributions of simultaneous calls.

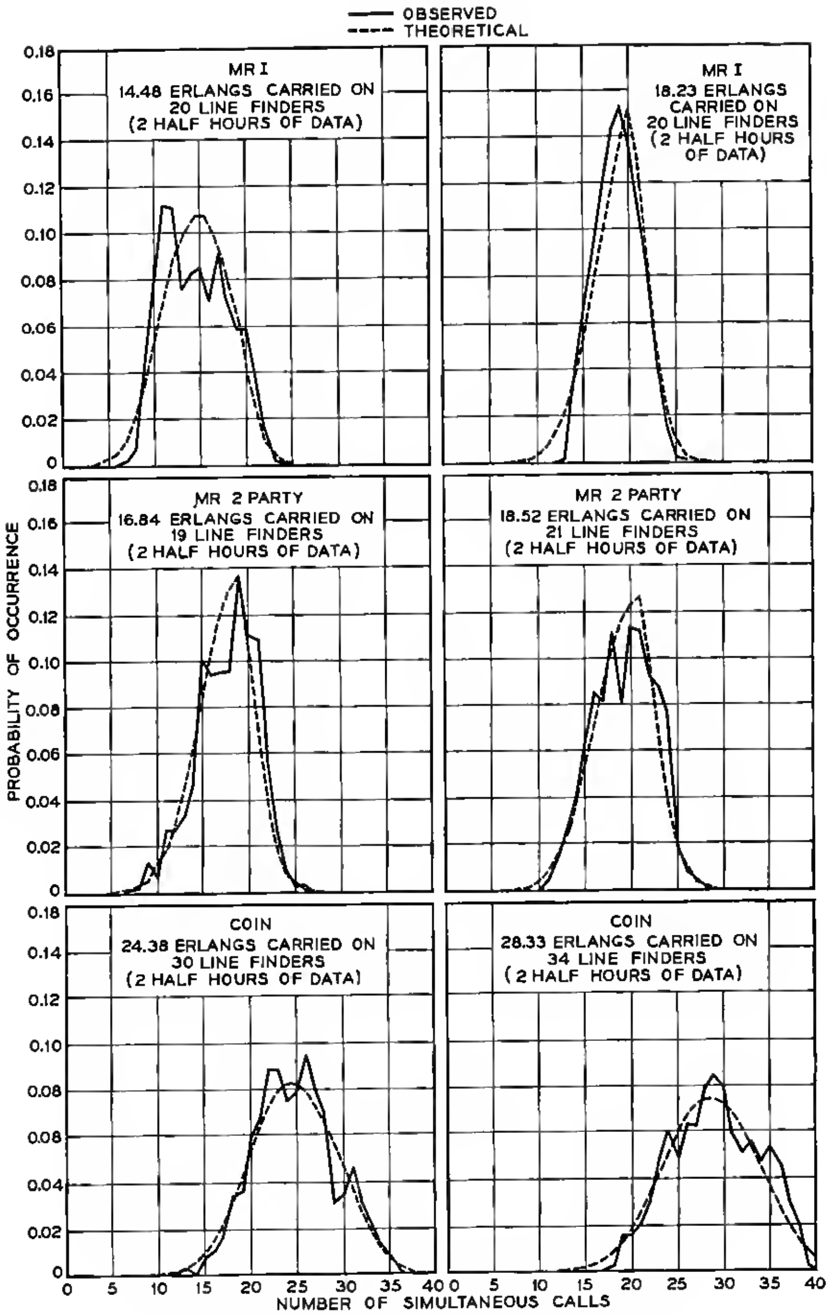

Fig. 11—Distributions of simultaneous calls.

The observer was provided with a means for introducing either no delay or one of four values of delay 2, 5, 10 or 15 seconds into the sender dial tone circuit. The observer took 50 observations using a particular value of dial tone delay and then shifted to another so that no particular value of delay would become evident to the customers during an afternoon's test. Each group of 50 observations comprised a mixture of message rate, coin and flat rate calls in the approximate proportions of 13 to 6 to 1, representing the respective volumes of traffic from these classes of service during the afternoon periods. It was not possible to distinguish PBX lines or two-party lines from the hulk of the message rate data nor PBX lines in the flat rate data, although to a limited extent the observer could identify PBX dialing by the generally faster pulsing. The coin data represent both public and semi-public customers.

Fig. 12 is a diagram for explaining the results shown on Figs. 13, 14 and 15 for the message rate, flat rate and coin classes of service, respectively as obtained with the sender monitoring circuit. Fig. 12 was obtained by the application of fitting curves to those message rate data of Fig. 13 for which a dial tone delay of five seconds was introduced by the observer. In the interval from $t = 0$ to $t = 5$ seconds, three curves A, B and C represent the per cent of subscribers still waiting at time t for dial tone. Curve A and its extension beyond $t = 5$ seconds represents the action of subscribers who would dial their calls before tone if dial tone were delayed indefinitely. Curve B and its extension beyond $t = 5$

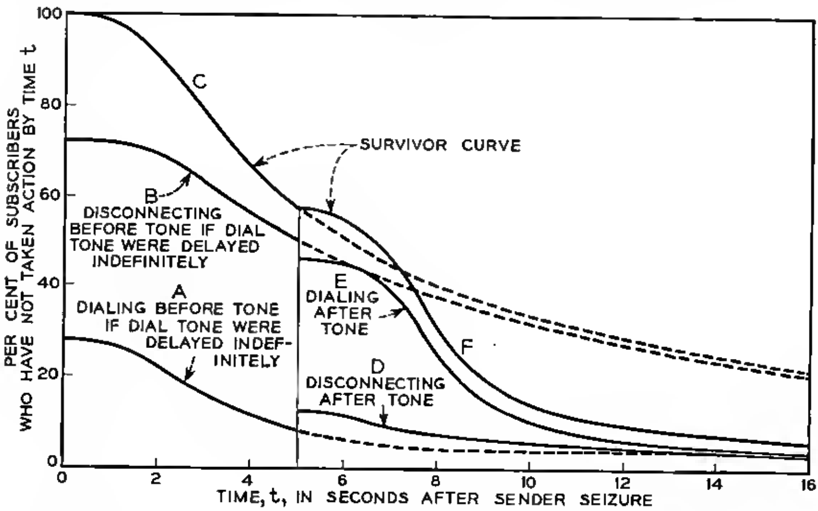

Fig. 12—Explanatory chart for sender monitoring observations; dial tone at $T = 5$ seconds.

seconds represents actions of subscribers who would disconnect if dial tone were delayed indefinitely. Curve C and its extension is the sum of the other two. Curve D in the region beyond $t = 5$ seconds represents the actions of subscribers who disconnect after tone, curve E represents the actions of subscribers who will dial their calls after tone and curve F represents the sum of the lower curves. Of interest is the fact that for an interval of about two seconds following dial tone (at $t = 5$ seconds), the observed total survivor curve F lies above the extended portion of

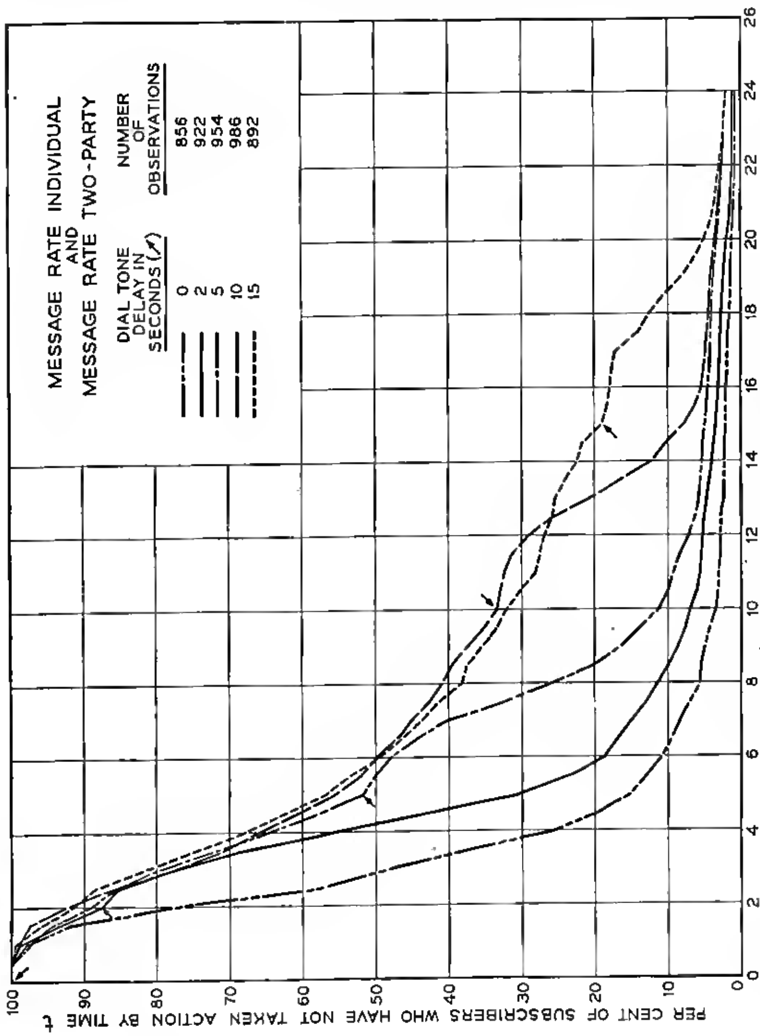

Fig. 13—Results of sender monitoring observations.

the hypothetical survivor curve C for infinitely delayed dial tone. This indicates that most of the subscribers who would have abandoned their attempts during this interval abruptly changed their minds and then consumed a noticeable interval of time after hearing tone before starting to dial. Thus, as might be anticipated, the subscribers exhibit a reaction time.

Fig. 13 shows the results in terms of survivor curves that were observed for the message rate class of service. Five sets of curves are

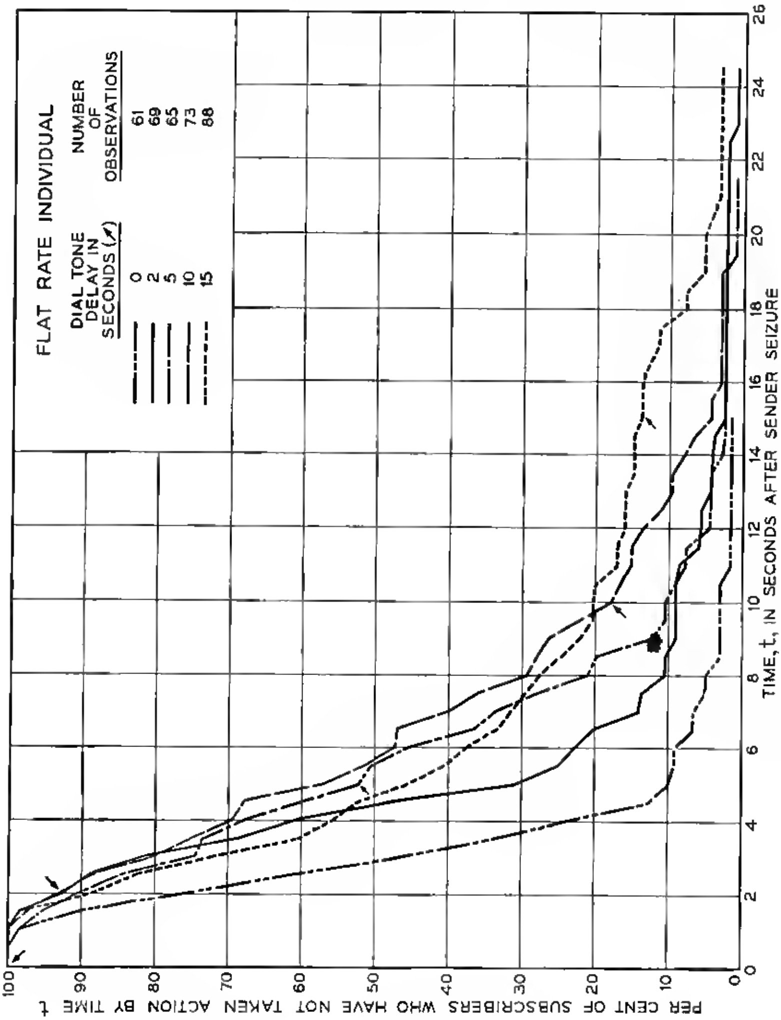

Fig. 14—Results of sender monitoring observations.

presented, namely for 0, 2, 5, 10 and 15 seconds of dial tone delays from the instant of sender seizure. It should be noted that the general contour of the various curves up to the receipt of dial tone and when extended beyond gives an estimate of the survivor curve for dial tone delayed indefinitely. Fig. 14 shows the results for the flat rate class of service (the data here are relatively meager), and Fig. 15 shows the results for coin customers.

Fig. 16 indicates for the three classes of service the progressive changes,

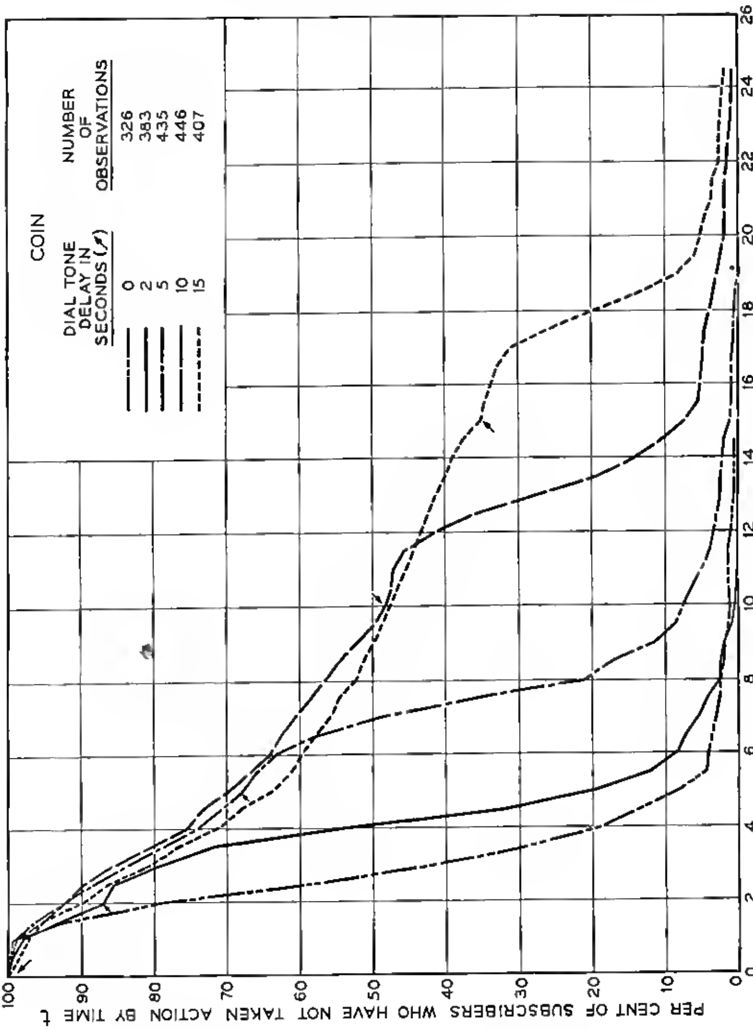

Fig. 15—Results of sender monitoring observations.

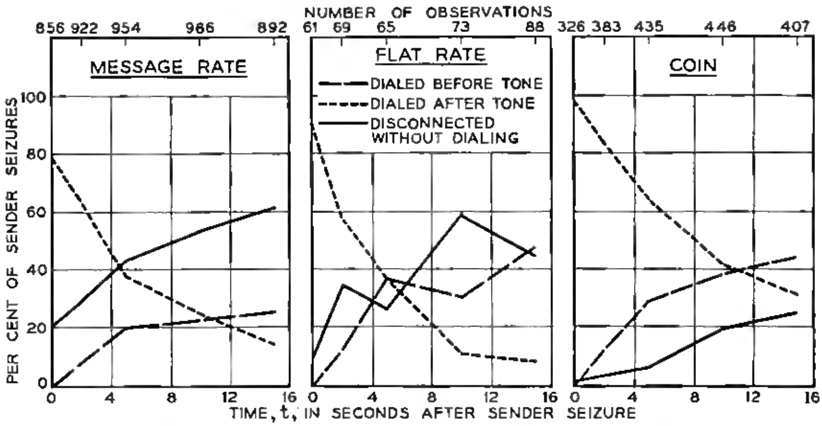

Fig. 16—Results of sender monitoring observations.

with increasing dial tone delay, of the per cent sender seizures resulting in dialing before tone, in dialing after tone and in disconnections for the five dial tone delay intervals studied.

Some general comments concerning Figs. 13 to 16 may be made.

1. There is a striking contrast between the message rate and coin classes of service. This may be due to the immediate financial stake that a coin customer has in his call. He is reluctant to disconnect before dial tone.

2. The results for the message rate and flat rate classes of service appear to be similar at the shorter dial tone delays; at the longer delays a higher proportion of flat rate subscribers have already dialed before dial tone. This apparent discrepancy may be due to tbe relatively small number of flat rate observations. Flat rate service in Sterling-3 and Main-2 was principally for professional people, such as doctors and nurses, who were thought to be more demanding than ordinary subscribers for prompt dial tone service. These results therefore should not be considered characteristic of flat rate customers generally.

3. Detailed analysis of the data (results not presented in this article) indicated that the distributions of time to first pulse for subscribers not observing tone, and for those waiting for tone, are quite similar for the three classes of service.

4. Only the unsmoothed raw data have been shown on Figs. 13 to 15 since certain inadequacies were detected in the observations. These were due to observer reactions and reading errors discovered as a result of comparing preliminary practice sender monitor test results with a simultaneous record obtained by the 100 pen tape recorder. The overall

effect is that the data obtained by the observer are generally displaced outward along the time axis by about 0.8 second.

5. The message rate data were for individual, PBX and two-party subscribers and the flat rate data were for individual and PBX subscribers. Furthermore, certain of the flat rate subscribers had auxiliary message rate service. It seems likely that different characteristics would be obtained for the individual, PBX and two-party subscribers since there appear to be reasons for expecting significant differences in their dialing habits. The PBX operator is in a position to "shop" for telephone service. If she fails to get dial tone on one outgoing line, she can try any other free line. This can also be done by subscribers with multiline service. This "shopping" for service tends to produce a large volume of disconnections when dial tone is slow. Tbe individual and the two-party subscribers cannot do this and hence they can be expected to show fewer disconnections.

6. The results are in terms of intervals of time from the instant the sender is seized. It would, of course, be preferable to have these results in terms of time from receiver off hook. Since on the average the sender is seized in a time interval of about the same magnitude as that of the reaction time of the observer, Figs. 13 to 15 can be read approximately correctly when the abscissas are redesignated "time in seconds from receiver off hook." The foregoing results have been presented to furnish an increased understanding of subscriber dialing habits. In the next section additional results based on individual line records taken on the tapes are presented.

SUBSCRIBER DIALING HABITS OBSERVED BY INDIVIDUAL LINE RECORDS

As indicated in the previous section, the results obtained by means of the sender monitor tests were subject to certain shortcomings, hence data taken on individual lines with the 100-pen recorder have been analyzed to augment the information concerning the dialing habits of subscribers.

As noted heretofore, several of the pens on the 100-pen recorder were available for taking observations on subscribers lines. Two pens were used per subscriber line, one recorded the operation of the subscriber's line relay while the other pen marked whenever the subscriber's line was busy. On an originating call both pens started marking when the subscriber initiated a call. When a line finder was obtained, the line relay pen ceased marking and it was presumed that the subscriber obtained dial tone at that instant. Dialing, hang-up and flashing by a

subscriber after receipt of dial tone are noted by breaks in the markings of the line busy pen. Dialing, hang-up or flashing before dial tone are noted by simultaneous breaks in the markings of both pens. Various intervals can be measured and the call attempts classified accordingly.

Observations were obtained in the foregoing manner during the course of the Sterling-3 line finder tests on the following numbers of subscriber lines:

Message rate—residential	87
—business	23
—PBX	12
—two party	32
Flat rate individual	7
Coin	21
	182

The observed data were classified for each of the above six types of subscribers in terms of the following categories:

1. Time to subscriber action before receipt of dial tone.
   a. Time from receiver off hook[9] to first digit dialed by subscriber.
   b. Time from receiver off hook to disconnecting action by tbe subscriber.
2. Time from receiver off hook to receipt of dial tone.
3. Time to subscriber action after receipt of dial tone.
   a. Time from receipt of dial tone to first digit dialed by subscriber.
   b. Time from receipt of dial tone to disconnecting action by tbe subscriber.

Because the data developed in this section are compared with botb the j factor analysis and the sender monitor test results and because tbe treatment of subscriber dialing habits before dial tone for eacb of tbese items is different, categories 1a and 1b are analyzed in two ways. In the j factor analysis, all actions of a subscriber prior to dial tone, except a disconnect, were considered to be one continuing demand for service. Thus for tbe first analysis, cases of dialing, flashing and short disconnections before tone lasting less tban three seconds were ignored. In the sender monitor tests the only items considered were the time to the first digit dialed by a subscriber and the time, if no dialing occurred-to the release of the sender by the subscriber. Thus for the second analysis, cases of dialing before tone and flashing or disconnections before tone

[9] When a customer initiates a call, the line relay operates. For individual line and two-party subscribers this occurs when the subscriber takes his receiver off the hook. For coin customers this occurs when the customer has taken his receiver off the hook and made a proper deposit. For a PBX line this occurs when the PBX attendant has established a connection to an outside line. All of this is collectively termed "receiver off hook."

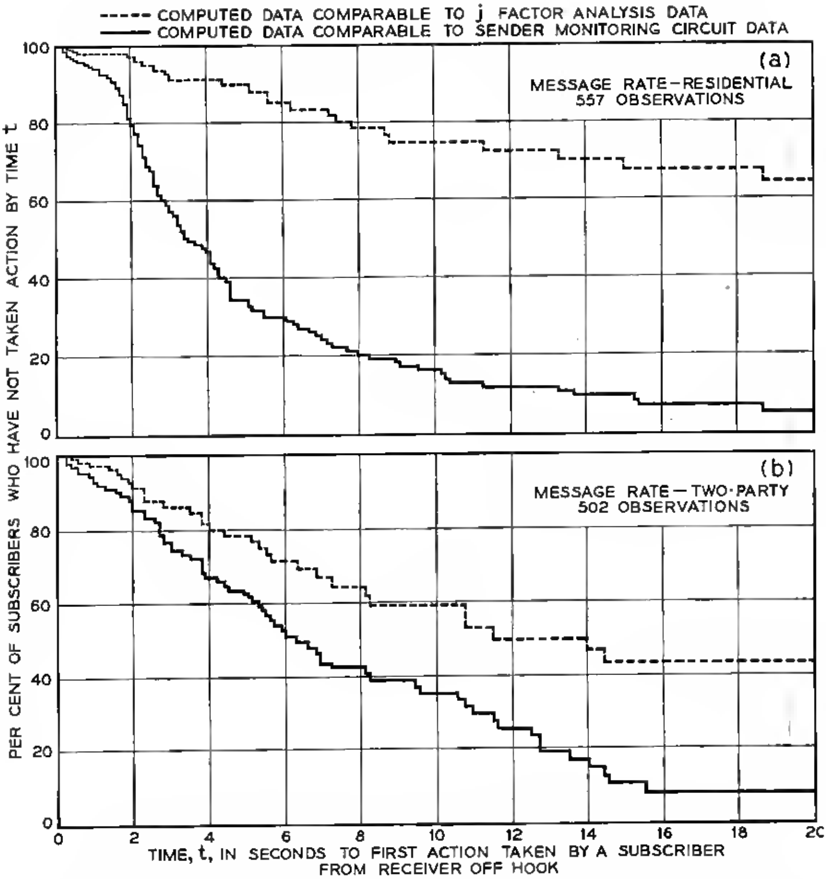

Fig. 17—Subscriber dialing habits based on individual line records when dia tone is delayed indefinitely.

that caused the sender to release were each counted as separate attempts. The results arrived at are shown as survivor curves on Figs. 17(a) to 19(b) for each of the six types of subscribers studied.

The survivor curves were developed by considering events during 0.1 second intervals. The number of cases of dialing before tone and the number of cases of disconnection before tone occurring during a 0.1 second interval were divided by the number of cases waiting for dial tone at the start of the interval. This ratio was considered to be a retirement rate. The complement of this rate gave a survival rate. By a progressive multiplication of survival rates from time, $t = 0$, the resulting survivor curves were obtained. Those cases receiving dial tone

during a particular 0.1 second interval are omitted from the number of cases waiting for dial tone at the start of the next interval.

In the development of the generalized trunking formula, the assumption was made that the waiting times of calls infinitely delayed have an exponential distribution. By assuming that the plots for the survivor curves in the development comparable to the j factor analysis are exponential distributions, it is possible by reading the value of t corresponding to 36.8 per cent of the subscribers still waiting for dial tone to obtain estimates of the values of H for the six types of subscribers. These estimates, most of which were obtained by extrapolation, are compared in

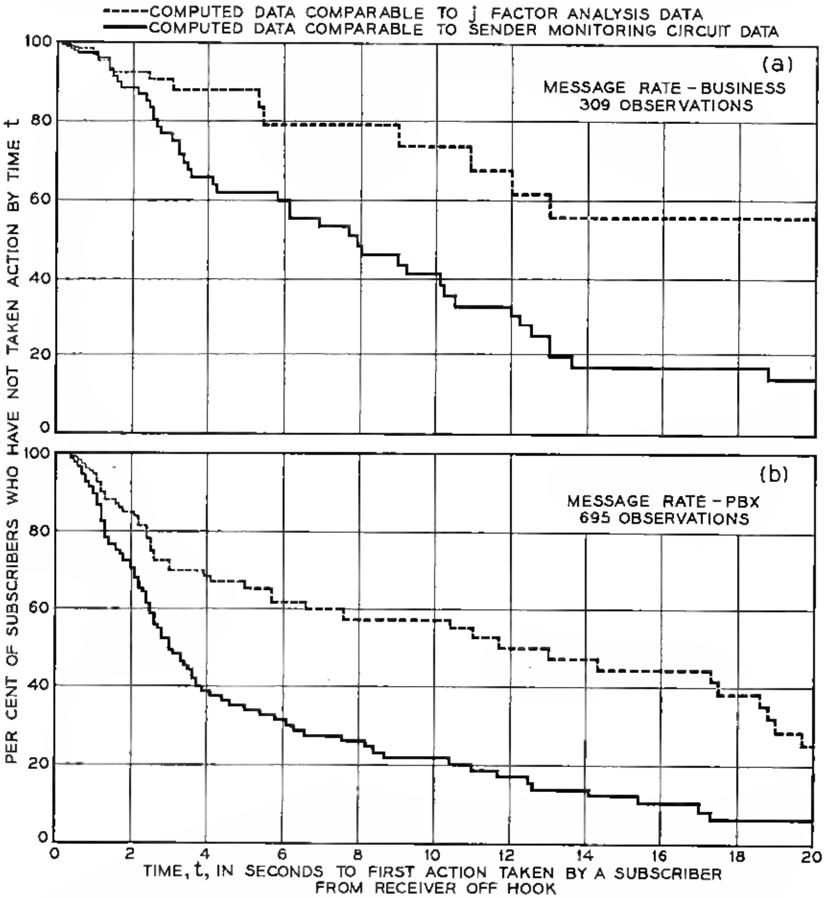

Fig. 18—Subscriber dialing habits based on individual line records when dial tone is delayed indefinitely.

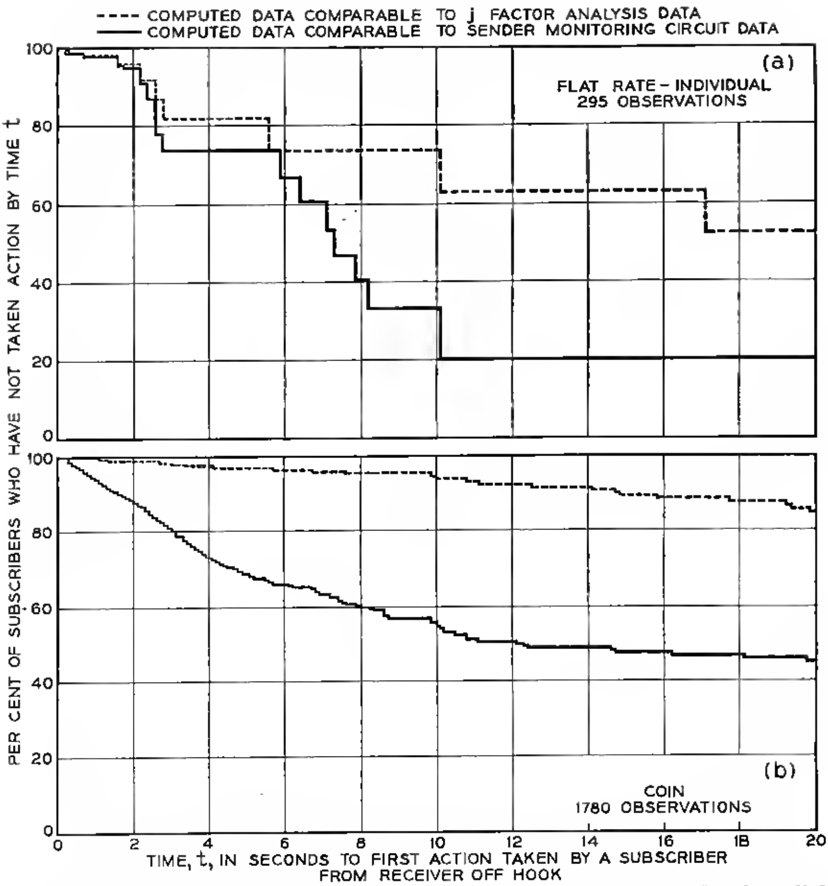

Fig. 19—Subscriber dialing habits based on individual line records when dial tone is delayed indefinitely.

Table IV with the values developed earlier in connection with the *j* factor analysis.

These results are not considered to be inconsistent since the tails of the survivor curves were constructed from very meager data. Conclusions based on Figs. 17(a) to 19(b) should therefore be regarded as having qualitative value only. The results principally indicate that the waiting-for-dial-tone characteristics of subscribers clearly vary with the different classes of service.

Comparisons between the lower survivor curves on Figs. 17(a) to 19(b) and the curves on Figs. 13 to 15 of the sender monitor tests are indicated by the percentages given in Table V of subscribers waiting for dial tone 5, 10 and 15 seconds from the time they requested service.

TABLE IV

	Estimated Values of H in Seconds	
	From figures 17(a) to 19(b)	From j factor analysis
Message rate—residential	45*	24
— business	33*	
— PBX	19	
— two-party	24*	42
Flat rate individual	32*	27
Coin	110*	74

* Rough extrapolated values

TABLE V

	Percentages of Subscribers Waiting for Dial Tone					
	Service Observation Figs. 17(a) to 19(b)			Sender Monitor Tests Figs. 13 to 15		
	5 secs	10 secs	15 secs	5 secs	10 secs	15 secs
Message rate						
Residential	34%	16%	10%	55%	33%	19%
Business	62	41	16			
PBX	35	22	13			
Two party	62	35	11			
Flat rate individual	74	34	20	55	19	14
Coin	68	55	47	67	48	35

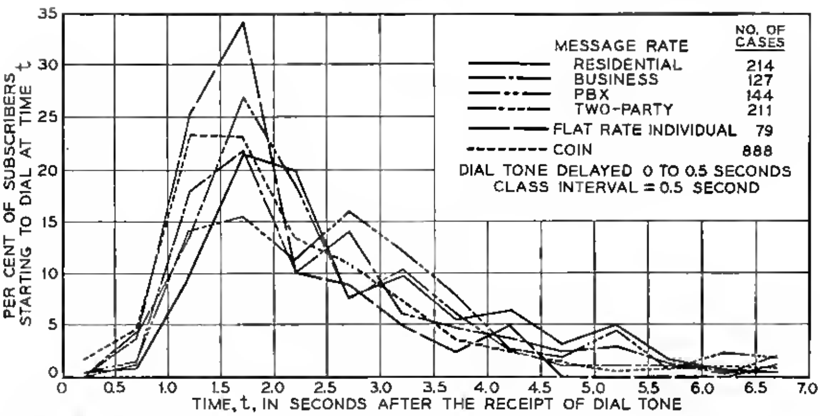

Fig. 20—Distributions of the start-to-dial times of subscribers who dial after the receipt of dial tone.

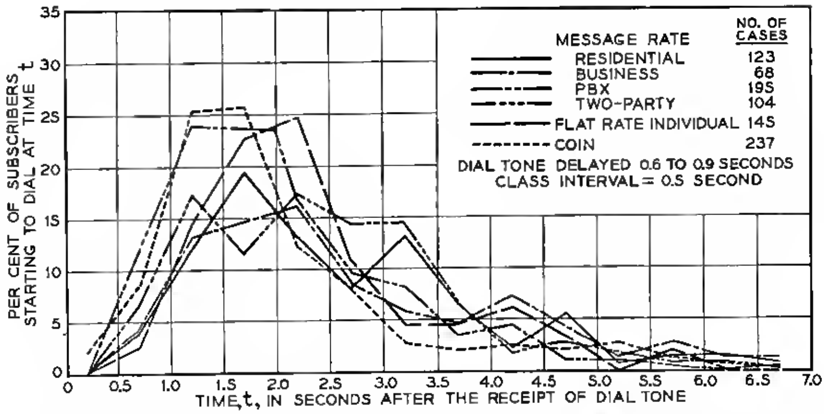

Fig. 21—Distributions of the start-to-dial times of subscribers who dial after the receipt of dial tone.

These results agree reasonably well when it is recalled that parts of the individual line data were scanty and that the sender monitor tests included the effects of observer reactions.

Once dial tone is received, it appears that all types of subscribers tend to follow a uniform dialing pattern. Figs. 20 to 23 show for a class interval of 0.5 second the distributions of the per cent of subscribers who dial at time t for the six types of subscribers studied. Figs. 20, 21 and 22 show the distributions when dial tone is received from 0.0 to 0.5, 0.6 to 0.9 and 1.0 to 1.9 seconds after dial tone, respectively. These curves

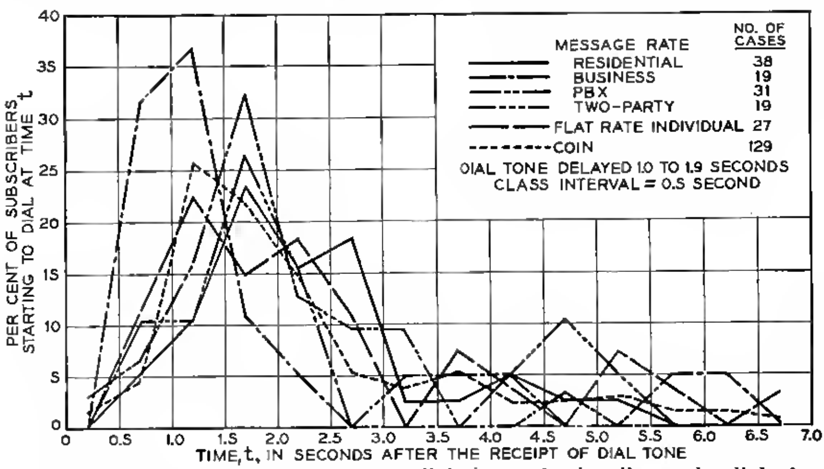

Fig. 22—Distributions of the start-to-dial times of subscribers who dial after the receipt of dial tone.

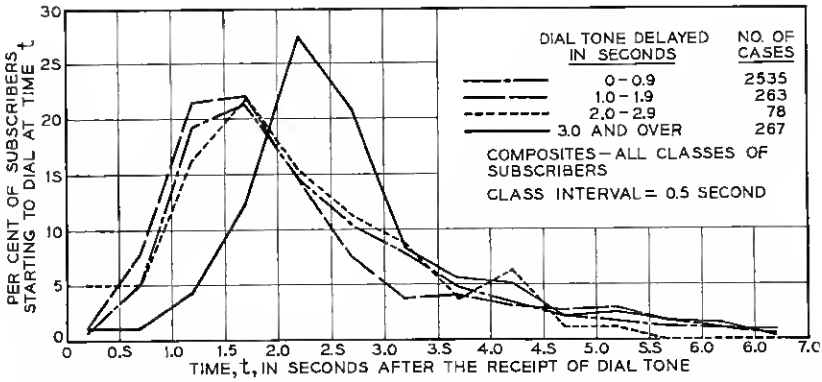

Fig. 23—Distributions of the start-to-dial times of subscribers who dial after the receipt of dial tone.

indicate that a strong similarity exists among the six types of subscribers with regard to their dialing patterns once dial tone is received.

Fig. 23 shows composite dialing distributions of the six types of subscribers for four dial tone delay intervals. For dial tone delays less than 3 seconds the dialing patterns are seen to be similar. Beyond 3 seconds delay the start-to-dial curve shifts outward, although the data are too scattered to indicate closely where the movement begins.

CONCLUSION

The foregoing report of the results of the tests conducted at the Sterling-3 central office indicates that the dialing habits of subscribers waiting for dial tone can be observed and analyzed to develop so-called j factors for use in a more general trunking formula than has been employed until recently in the Bell System. The report also presents descriptive data regarding the patterns subscribers follow when waiting for dial tone.

ACKNOWLEDGEMENT

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